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Holography with Ramond-Ramond fluxes

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Abstract: Starting from the non-linear sigma model of the IIB string in the light-cone gauge, we analyze the role of RR fluxes in Holography. We find that the worldsheet theory of states with only left or right moving modes does not see the presence of RR fields threading a geometry. We use this significant simplification to compute part of the strong coupling spectrum of the two dimensional NCOS theory. We also reproduce the action of a closed string in a PP-wave background using this general formalism; and we argue for various strategies to find new systems where the closed string theory may be exactly solvable.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, Non-Commutative Geometry.

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1. Introduction

One of the central themes in string theory during recent years is a tantalizing correspondence between closed and open string dynamics [1]–[4]. This duality is realized in a myriad of different flavors that yet share certain general commonalities. At low energies, the duality often asserts a holographic map between gravitational dynamics and certain non-gravitational theories. The correspondence however necessarily and generally involves the full closed string theory instead of a low energy truncation to Einstein gravity. Yet technical problems prevent one from exploring this holographic duality in its full form. One of these problems has to do with the fact that, in many settings, the closed string dynamics in question unfolds in the presence of Ramond-Ramond (RR) fluxes. Understanding the effects of these fluxes on closed string propagation is hence of importance to unravelling the duality. This issue is of particular interest specially when the dual non-gravitational theory is a non-commutative open string (NCOS) theory [5]–[8].[1] In such scenarios, the holographic duality becomes a map between two two-dimensional worldsheet theories. We may hope that understanding this map would teach us fundamental lessons about the mechanism underlying Holography.

In this work, we will try to take the first steps in exploring closed string propagation in RR fluxes. We confine our discussion to IIB string theory and, after establishing a certain general formalism, we will focus on the case involving the two dimensional NCOS theory and the case of $AdS_5 \times S^5$.

[1] Note that the compactified NCOS theory has also a sector of newtonian gravity [9]-[11].

The full light-cone action of IIB string theory in backgrounds of interest was derived in [12]. We rewrite here the results in a somewhat more conventional notation. We separate the string action into its kinetic piece and two parts labeled by the number of spinor fields they entail:

$$\mathcal{I} = \mathcal{I}^{kin} + \mathcal{I}^{(2)} + \mathcal{I}^{(4)} . \tag{1.1}$$

The kinetic piece is given by

$$\begin{aligned}\mathcal{I}^{kin} \equiv \frac{1}{2\pi\alpha'} \int d\tau d\sigma \Big(& 2\sqrt{-h}\; h^{ij} V_i^+ V_j^- - 4\, \varepsilon^{ij} V_i^+ V_j^- B^{(1)-+} + \frac{1}{2}\sqrt{-h}\; h^{ij} V_i^a V_{ja} - \\ & - \frac{1}{2}\varepsilon^{ij} V_i^a V_j^b B_{ab}^{(1)} - 2\, i\, V_i^+ \sqrt{-h}\; h^{ij} \bar\theta \rho^0 \partial_j \theta + 2\, i\, V_i^+ \varepsilon^{ij} \bar\theta \rho^1 \partial_j \theta \Big) .\end{aligned} \tag{1.2}$$

We have defined

$$V_i^{\pm} \equiv \partial_i x^m e_m^{\pm} , \tag{1.3}$$

where e_m^a is the vielbein; and $B_{ab}^{(1)}$ is the NSNS B-field. Spacetime indices are labeled by $m, n, \ldots$, while tangent space indices are written as $a, b, \ldots$. The light-cone direction is defined as follows

$$e_m^{\pm} = \frac{1}{2}\left(e_m^0 \pm e_m^d \right) , \tag{1.4}$$

with d being an arbitrarily chosen space direction and 0 denoting the tangent space time index. Throughout, tangent space indices $a, b, \cdots$ run only over the eight directions transverse to the light-cone; hence, we have $a, b, \ldots = 1$ to 9 except d. The indices i and j will always refer to the worldsheet coordinates τ and σ. Correspondingly, h^{ij} is the worldsheet metric, and $\varepsilon^{\tau\sigma} = -\varepsilon^{\sigma\tau} = 1$. Two spinors θ_A, with $A = 1, 2$, are sixteen component ten dimensional Majorana-Weyl spinors of the same chirality, and are collected into a doublet $\theta \equiv (\theta_1, \theta_2)$. The matrices ρ^i act in this two dimensional space and may be viewed as defining a worldsheet spinor representation for these fermions. More details about the spinor representation we are using may be found in appendix A and [12].

The additional pieces of the action involve: 'mass terms' for the fermions that we formally write as

$$\mathcal{I}^{(2)} \equiv \frac{1}{2\pi\alpha'} \int d\tau d\sigma\; V_i^+ V_j^a \left(\sqrt{-h}\; h^{ij} \mathcal{A}_a + \varepsilon^{ij} \mathcal{B}_a \right) ; \tag{1.5}$$

$\mathcal{A}_a$ and $\mathcal{B}_a$ are quadratic in θ; and interaction terms quartic in the spinors given by

$$\mathcal{I}^{(4)} \equiv \frac{1}{2\pi\alpha'} \int d\tau d\sigma\; \sqrt{-h}\; h^{ij} V_i^+ V_j^+ \mathcal{C} , \tag{1.6}$$

with

$$\mathcal{C} \equiv \mathcal{T}^{ab}{}_{ab} \mathcal{P} + \mathcal{T}^{ca}{}_c{}^b \mathcal{Q}_{ab}^{(1)} + \mathcal{S}^{ca}{}_c{}^b \mathcal{Q}_{ab}^{(2)} + \mathcal{T}^{abcd} \mathcal{R}_{abcd}^{(1)} + \mathcal{S}^{abcd} \mathcal{R}_{abcd}^{(2)} . \tag{1.7}$$

Two types of quartic terms of interest are involved and are collected into the following combinations[2]

$$\begin{aligned} \mathcal{T}^{abcd} &\equiv (\bar\theta\sigma^{ab}\theta)(\bar\theta\sigma^{cd}\theta) + (\bar\theta\rho^1\sigma^{ab}\theta)(\bar\theta\rho^1\sigma^{cd}\theta) ; \\ \mathcal{S}^{abcd} &\equiv i(\bar\theta\rho^1\sigma^{ab}\theta)(\bar\theta\sigma^{cd}\theta) - i(\bar\theta\sigma^{ab}\theta)(\bar\theta\rho^1\sigma^{cd}\theta) . \end{aligned} \tag{1.8}$$

[2]Note that our notation for spinors defers from [12]. In particular, $\theta_{old} = \theta_1 + i\theta_2$ and θ is now the doublet $\theta \equiv (\theta_1, \theta_2)$. See appendix A for more details.

These objects have the following symmetries in their index structures

$$\mathcal{T}^{abcd} = \mathcal{T}^{cdab} = -\mathcal{T}^{bacd} = -\mathcal{T}^{abdc}\,; \tag{1.9}$$

and

$$\mathcal{S}^{abcd} = -\mathcal{S}^{cdab} = -\mathcal{S}^{bacd} = -\mathcal{S}^{abdc}\,. \tag{1.10}$$

In these expressions, $\mathcal{A}_a$, $\mathcal{B}_a$, $\mathcal{P}$, $\mathcal{Q}^{(1)}_{ab}$, $\mathcal{Q}^{(2)}_{ab}$, $\mathcal{R}^{(1)}_{abcd}$ and $\mathcal{R}^{(2)}_{abcd}$ contain the background supergravity fields in rather elaborate combinations. These were derived in [12] starting from the superspace formalism, and are reproduced in appendix A in a new more useful notation.[3]

The action (1.1) may be trusted (figuratively speaking) as long as the following conditions are satisfied [12]: (1) All non-bosonic background fields are zero; (2) All background fields are independent of the light-cone coordinates; (3) The index structure of the background fields is such that the light-cone directions '$-$' and '$+$' always appear in pairs, if at all; (4) And the string frame metric may be cast into a diagonal form. These conditions are satisfied by most supergravity solutions describing various configurations of Dp branes. Generically, the light-cone direction is to be chosen such that it is parallel to the worldvolume of one of the D-branes in a given system. Hence, propagation of closed strings in the vicinity of D-branes may readily be studied using this action. Most of the interaction terms that appear in (1.1) have never been previously explored. Generically, the implied dynamics is very complex. Yet, certain aspects, particularly ones relevant to understanding the holographic duality, may still be unravelled using various approximation techniques and special settings.

Before going into any particulars, let us make a few general comments about the structure of our action. An issue of paramount importance is whether the presence of quartic terms in the spinors can result in shifting the standard fermionic vacuum $\langle\theta\rangle = 0$ to a non-zero value. We may expect that in certain situations, the spinor fields may develop a condensate, much like, for example, in the Gross-Neveu model [13]. Assuming that this is the case, we can easily see that the dynamics of the bosonic fields x^m describing the embedding of the closed string in the given background can change dramatically. The fermionic pieces of the action would indirectly play the role of sources to the V_j^a field in the worldsheet theory. Through such a mechanism, RR fields can affect even the leading classical propagation (i.e. the "ground state") of a closed string which is probing a D-brane geometry.

There are two main results in this work that can be summarized as follows. For closed string states with only left or right moving modes on the worldsheet — and in particular for the case of center of mass motion — we find that the couplings of the RR fluxes to the spinors cancel. This can already be seen from the form of our action: Looking at (1.5)

[3] In passing, let us observe that this action is written by fixing the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of the IIB supergravity theory [12]. As a result, the S-duality group is broken to a subset given by transformations of the form

$$\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}. \tag{1.11}$$

The remnant symmetry corresponds to shifting the RR axion by b.

and (1.6), with the relations $V_\sigma^+ = \pm V_\tau^+$ and $V_\sigma^a = \pm V_\tau^a$, and the gauge choice $h^{\tau\tau} = 1$ and $h^{\sigma\sigma} = -1$, it can be observed that both of these pieces vanish. The consistency of these conditions and gauge choice with the worldsheet dynamics is shown in section 2. This implies that the complications having to do with a non-trivial vacuum for the spinors may be circumvented and the dynamics is determined from the bosonic part of the action.[4] We use this conclusion to compute part of the strong-coupling spectrum of the two dimensional NCOS theory on a circle. The results agree with those of [14], but are now presented in the light-cone gauge with rigorous justification for the needed assumptions. We then look at the action (1.1) for $AdS_5 \times S^5$ backgrounds and we take the PP-wave limit on the worldsheet theory. We argue that dimensional analysis and the general form of the action immediately imply that quartic parts in (1.6) are subleading to the kinetic piece by two powers of the AdS length scale. This general approach can be used to look for other interesting scaling regimes in different background geometries.

The outline of the paper is as follows. In section 2, we present the classical equations of motion of the closed string. We distinguish two cases: backgrounds with or without an NSNS B-field, yet involving RR fluxes. In section 3, we apply our analysis to the case corresponding to a holographic duality between two dimensional NCOS theory on a circle and closed strings in a background geometry involving both NSNS and RR fluxes. In section 4, we present an academic exercise in applying the technology to the much studied $AdS_5 \times S^5$ system [15]-[17]; and proceed to take the PP-wave limit [18]-[23] to reproduce well-known results that illustrate another mechanism to eliminate the non-linearities in the action. In section 5, we comment on future directions involving different scenarios, such as integrable worldsheet theories and new and special string backgrounds.

2. Closed strings and RR fluxes

In studying the classical dynamics implied by (1.1), we need to subject the system to the constraints

$$T_{ij} \equiv \frac{1}{\sqrt{-h}} \frac{\delta \mathcal{I}}{\delta h^{ij}} = 0 \,. \tag{2.1}$$

Our action is endowed with two dimensional reparameterization symmetry and scale invariance. These allow us to fix the worldsheet metric h_{ij}. In general backgrounds, we need to be careful about this step that is often taken for granted (see for example [15]–[17]). The equation of motion for the V^- field is given by

$$\delta_{V^-} \mathcal{I} = \delta_{V^-} \mathcal{I}^{kin} = 0 \,. \tag{2.2}$$

For concreteness, let us denote the light-cone directions by $\{t, y\}$ with corresponding tangent space labels $\{0, 1\}$. Note that these are isometry directions for our background by construction. We assume that it is possible to choose coordinates such that

$$e_t^0 = e_y^1 \equiv l \Rightarrow ds^2 = l^2 \; dt^2 - l^2 \; dy^2 + \cdots \,. \tag{2.3}$$

[4] Note also that this point is consistent with using perturbations in low energy supergravity to explore the holographic duality as these typically correspond to unexcited states of the closed string.

l may be a function of all the coordinates except t and y. Assumption (2.3) is not truly needed, but makes the discussion of the NCOS scenario later notationally more transparent. We then have

$$V_i^{\pm} = \frac{l}{2}\partial_i\left(t \pm y\right) \equiv l\ \partial_i y^{\pm}\,, \tag{2.4}$$

which is our definition for $y^{\pm}$ throughout. And equation (2.2) becomes (using periodic boundary conditions for closed strings)

$$\partial_j\left(\sqrt{-h}\ h^{ij} l^2 \partial_i y^+ - 2\varepsilon^{ij} l^2 \partial_i y^+ B^{(1)-+}\right) = 0\,. \tag{2.5}$$

We now need to distinguish two cases: one involving backgrounds without an NSNS B-field; and one that involves a non-zero $B^{(1)-+}$.

2.1 Zero NSNS B-field in the light-cone direction

Let us first assume that we are dealing with a situation where $B^{(1)-+} = 0$. It is then easy to see that the following conditions

$$\sqrt{-h}\ h^{\tau\tau} = \frac{1}{l^2}\,, \qquad \sqrt{-h}\ h^{\sigma\sigma} = -l^2\,, \qquad \sqrt{-h}\ h^{\tau\sigma} = 0\,; \tag{2.6}$$

$$y^+ = p^+\tau \quad \Rightarrow \quad V_\tau^+ = l\ p^+\,, \qquad V_\sigma^+ = 0\,. \tag{2.7}$$

are consistent with (2.5) and fix all the conformal symmetry on the worldsheet. Equation (2.7) plays the standard role of eliminating the residual symmetry that is still available after requiring (2.6). Substituting these in (1.1), and rescaling the spinors as in

$$\sqrt{\frac{p^+}{l}}\theta \to \theta\,, \tag{2.8}$$

we are lead to the action

$$\begin{aligned}\mathcal{I} = \frac{1}{2\pi\alpha'}\int d\tau d\sigma\Bigg[&-2\ i\ \bar{\theta}\rho^0\partial_\tau\theta + 2\ i\ l^2\bar{\theta}\rho^1\partial_\sigma\theta + 2p^+\partial_\tau y^- + \\ &+ \frac{1}{2}\left(\frac{e_m^a e_{na}}{l^2}\partial_\tau x^m \partial_\tau x^n - l^2 e_m^a e_{na}\partial_\sigma x^m \partial_\sigma x^n\right) + \\ &+ \partial_\tau x^m \mathcal{A}_a e_m^a + l^2 \partial_\sigma x^m \mathcal{B}_a e_m^a + l^2\mathcal{C}\Bigg]\,,\end{aligned} \tag{2.9}$$

which has properly normalized kinetic terms for the fermions[5]. The constraints (2.1) become the two statements

$$\begin{aligned}p^+\left(\partial_\tau \pm \partial_\sigma\right) y^- = &\ i\left(\bar{\theta}\rho^0\partial_\tau\theta \pm \bar{\theta}\rho^0\partial_\sigma\theta\right) - \frac{1}{2}l^2\mathcal{C} - \\ &- \frac{1}{2}\left(\partial_\tau x^m \pm \partial_\sigma x^m\right)\mathcal{A}_a e_m^a - \frac{1}{4l^2}e_m^a e_{na}\partial_\tau x^m \partial_\tau x^n - \\ &- \frac{l^2}{4}e_m^a e_{na}\partial_\sigma x^m \partial_\sigma x^n \mp \frac{1}{2l^2}e_m^a e_{na}\partial_\tau x^m \partial_\sigma x^n\,.\end{aligned} \tag{2.10}$$

[5]Note that the rescaling (2.8) does not introduce additional terms into the action involving derivatives of l.

Note that in these two equations, the vielbein appears explicitly, instead of the metric. We choose this notation to emphasize that the indices m and n in spacetime are restricted to be summed over only the eight directions transverse to the light-cone.

In general, equations (2.9) and (2.10) describe a complicated system. We could choose an ansatz for which the quadratic terms in the spinors vanish. But with the presence of quartic interactions, we should generically expect that mass terms are generated at the quantum level and a spinor condensate may develop. Alternatively, we may focus on BPS-like states of the closed string, with only left or right moving modes. This would result in a significant simplification. To make this aspect transparent, the formalism requires choosing a slightly different and unconventional gauge on the worldsheet. We will present this in the next subsection, where we will be able to consider the case with non-zero $B^{(1)-+}$ in the same setting. Beyond this, taming equations (2.9) and (2.10) into ones that are computationally manageable involves either restricting oneself to special backgrounds, either from the outset or through a judicious scaling regime as in the case of PP-waves geometries; or being lucky enough that the corresponding non-linear theory happens to be integrable. Alternatively, one can turn around the argument and look for the proper conditions on the background fields so as to make headway on the problem. We will comment on these possibilities in section 4 and the discussion in section 5. For now, we move onto a more interesting scenario.

2.2 Nonzero $B^{(1)-+}$

Consider next the situation where $B^{(1)-+}$ is not necessarily zero; i.e. we have a non-zero NSNS B-field parallel to the light-cone. Equation (2.5) may then be solved if

$$\sqrt{-h}\; h^{\tau\tau} = +1\,, \qquad \sqrt{-h}\; h^{\sigma\sigma} = -1\,, \qquad \sqrt{-h}\; h^{\tau\sigma} = 0\,, \tag{2.11}$$

and

$$\partial_{\mp} y^{+} \equiv (\partial_\tau \mp \partial_\sigma)\, y^{+} = 0\,, \qquad e^{a}_{m} \partial_{\mp} x^{m} = 0\,. \tag{2.12}$$

This means that we allow only for either left moving or right moving modes on the closed string, *but not both.* It implies that for all background fields, we have

$$\partial_{\mp}\,(\text{fields}) = 0\,. \tag{2.13}$$

However, we still have $\partial_{\mp} y^{-} \neq 0$. Note that, at this stage, we have fixed the reparametrization symmetry in the standard way by choosing a worldsheet metric; *and* half of the residual conformal symmetry: $y^{+}(\sigma^{\pm})$ is still free to be fixed.

All of this formalism goes through as well for the case where the B-field is zero. The difference is that, when $B^{(1)-+} \neq 0$, this is the only solution we are able to easily write. It is a mildly entertaining fact that this reduction in the degrees of freedom makes it 'more likely' for a closed string to mimic open string dynamics. On an open string, right and left moving modes are correlated; on our closed string, either right or left moving modes are being allowed. And indeed, it is when the B-field is turned on in the time direction that the dual theory is more than a field theory, but a full (non-commutative) open string theory. Yet the solutions given by (2.11) and (2.12) are not the only possible ones. Ours is a condition which is the analogue of the BPS condition for a closed string in flat space. We focus on it since it can describe the center of mass motion of the closed string. However, we observe that this discussion suggests that there may be a dynamical mechanism, summarized by (2.5), which results in correlating the right and left moving modes on the closed string in the presence of a B-field for the most general solution to (2.5).

We now proceed to identifying the light-cone momentum by using

$$\frac{p^+}{\Sigma} \equiv \frac{\partial \mathcal{I}}{\partial \partial_\tau y^-} \,. \tag{2.14}$$

We are assuming for convenience that the y direction is compact of size Σ; and we are using this length scale to measure light-cone momentum. This yields to

$$2\pi\alpha' \frac{p^+}{2\Sigma} = \left(l^2 \mp B^{(1)}_{ty}\right) \partial_\tau y^+ \,, \tag{2.15}$$

with $2B^{-+} = -B_{ty}/l^2$. Equation (2.15) is the analogue of $y^+ = p^+\tau$ of the previous subsection. We have now fixed all of the residual worldsheet symmetries. We summarize these statements again:

$$\partial_\mp y^+ = 0 \,, \qquad 2\pi\alpha' \frac{p^+}{\Sigma} = \left(l^2 \mp B^{(1)}_{ty}\right) \partial_\pm y^+ \,. \tag{2.16}$$

We then have

$$V^+_\tau = 2\pi\alpha' \frac{p^+}{2\Sigma} \left(l \mp \frac{B^{(1)}_{ty}}{l}\right)^{-1} \,; \tag{2.17}$$

and

$$V^+_\sigma = \pm V^+_\tau \,. \tag{2.18}$$

We now substitute all these in (1.1), and, after rescaling the spinors as in

$$\sqrt{\frac{p^+}{\Sigma}} \left(l \mp \frac{B_{ty}}{l}\right)^{-1/2} \theta \to \theta \,, \tag{2.19}$$

we obtain the action

$$\begin{aligned}\mathcal{I} = \int d\tau d\sigma \; & \frac{p^+}{\Sigma} \partial_\mp y^- - i\bar{\theta}\left(\rho^0 \pm \rho^1\right) \partial_\pm \theta + \\ & + \frac{1}{2\pi\alpha'} \left[\frac{1}{2} e^a_m e_{na} \partial_+ x^m \partial_- x^n - B^{(1)}_{ab} \partial_\tau x^m \partial_\sigma x^n e^a_m e^b_n\right] .\end{aligned} \tag{2.20}$$

The upper/lower choices of the sign correspond to the possibilities of allowing either left or right moving modes. Note that only one of the two fermions has a kinetic term (see the definitions of the ρ matrices in appendix A; the combination $\rho^0 \pm \rho^1$ is a projection operator that picks either spinor θ_2 or spinor θ_1). The constraints (2.1) become

$$\begin{aligned}\frac{p^+}{2\Sigma} \frac{l^2}{l^2 \mp B^{(1)}_{ty}} \partial_\pm y^- + \frac{1}{8\pi\alpha'} e^a_m e_{na} \left(\partial_\tau x^m \partial_\tau x^n + \partial_\sigma x^m \partial_\sigma x^n\right) + \\ + \frac{1}{4} 2\pi\alpha' \mathcal{C} + \frac{1}{4} \mathcal{A}_a e^a_m \partial_\pm x^m - \frac{i}{2} \bar{\theta} \rho^0 \partial_\pm \theta = 0 \,.\end{aligned} \tag{2.21}$$

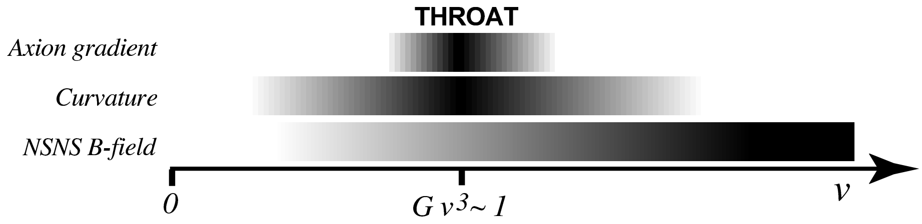

Figure 1: The main features of the NCOS geometry. Darker regions correspond to higher values.

Note again that m and n are spacetime indices transverse to the light-cone. We are then left with a single constraint (one for each choice of sign) since

$$T_{00} = 0 \qquad \text{and} \qquad V_\tau^a \mp V_\sigma^a = 0 \quad \Rightarrow \quad T_{01} = 0\,. \tag{2.22}$$

The interesting fact is that the quartic interaction terms in $\mathcal{I}^{(4)}$ now appear only in the constraint. The evolution is determined by the action (2.20), which must preserve the constraint. Hence, if we were to setup initial conditions consistent with the constraint (2.21), its stability under time evolution is up to (2.20). This implies that the vacuum

$$\langle\theta\rangle = 0\,, \tag{2.23}$$

taken as a consistent ansatz with the constraint (2.21) for a class of closed string trajectories, is a stable vacuum.

We next introduce the light-cone energy E_{LC}

$$\Pi_{y^+} \equiv \frac{\partial \mathcal{L}}{\partial\partial_\tau y^+} = l^2\left(1 - 2B^{(1)-+}\right)\frac{\partial_+ y^-}{2\pi\alpha'} + l^2\left(1 + 2B^{(1)-+}\right)\frac{\partial_- y^-}{2\pi\alpha'} \equiv \frac{E_{LC}}{\Sigma} + F(\tau,\sigma)\,, \tag{2.24}$$

where we have used $\langle\theta\rangle = 0$. Center of mass motion would correspond to $F(\tau,\sigma) = 0$. And the constraint (2.21) is the mass-shell condition once one solves (2.24) for $\partial_\pm y^-$ and substitutes in (2.21). Next, we apply this formalism to the NCOS background, focusing only on center of mass motion of a wound closed string.

3. The case of two dimensional NCOS revisited

The holographic duality of interest to us is the one considered in [14]. We have a bound state system of N IIB strings and M D-strings; and we consider the decoupling scaling limit. The corresponding geometry is given in appendix B and involves a non-zero B-field as well as RR gauge fields; one for the IIB axion, and a constant one from the D-strings. We denote by t and y the directions parallel to the worldvolume of the bound system, and we use v to denote the radial direction, which is identified with energy scale in the dual theory. The geometry involves an interesting throat region at $Gv^3 \sim 1$. The rest of the space is a seven-sphere of size varying with v. The dual theory is two dimensional NCOS theory with string scale $1/(2\pi\alpha_e) = 1/(2\pi {l_e}^2)$, coupling constant G, and D-string charge M. The relevant features of the background geometry are depicted in figure 1.

In [14], the center of mass motion of a closed string wound in the y direction was studied after assuming that the RR fields may be ignored. We saw in the previous section that this assumption is indeed justified. The dynamics explored in [14] involved, in particular, bounded trajectories that were used to predict part of the strong coupling spectrum of the NCOS theory. Having validated the assumptions of [14], we now proceed to reproduce some of the results, now in the light-cone formalism introduced here. We will find that the physical conclusions of [14] are not affected by our choice of a different gauge which happens to be more useful for understanding the irrelevance of the RR fields to the center of mass motion. In this regards, we will not present all the details of the NCOS theory and refer instead the reader to [14] for a more complete exposition to the subject.

We consider a closed string wound in the light-cone direction y radially infalling towards $v = 0$. Given that one is allowed left or right moving modes only, we need to choose the light-cone momentum in such a way as to correlate it with the desired winding number if we do not desire to consider the case with additional momentum along the closed string. Applying $\partial_\mp$ to the constraint equation (2.21), we find that $y^- = f_1(\sigma^+)+f_2(\sigma^-)$. Applying $\partial_\mp$ to (2.24) with $F(\tau,\sigma) = 0$ (i.e. the case of center of mass dynamics), we find that $\partial^2_\mp y^- = 0$. Hence, we write

$$y^- = -\frac{2K}{\Sigma}\sigma^\mp + g(\sigma^\pm)\,. \tag{3.1}$$

The two choices of signs we have for left or right moving modes translate now to two possible orientations for the winding: parallel or anti-parallel to the B-field. We will map the constant K onto winding number soon. And $g(\sigma^\pm)$ is fixed by (2.24).

For center of mass motion, the classical dynamics is entirely determined by the constraint [14]. This now becomes the statement

$$\frac{1}{8}g_{vv}\left(\partial_\pm v\right)^2 + \frac{p^+}{2\Sigma}\left((2\pi\alpha')^2\frac{E_{LC}}{\Sigma}\frac{l^2}{l^4 - B^{(1)\,2}_{ty}} + (2\pi\alpha')\frac{l^2}{l^2 \pm B^{(1)}_{ty}}\frac{2K}{\Sigma}\right) = 0\,, \tag{3.2}$$

where we have used the fermion vacuum $\langle\theta\rangle = 0$. Denoting the winding number of the closed string along the y direction by $\omega > 0$, while keeping track of orientation through the two choices of signs, we must then have

$$K = -\frac{\omega}{2}\Sigma\,. \tag{3.3}$$

The factor of 2 arises from the definition of the light-cone directions $y^\pm$, (3.1), and by demanding $y \sim y + \Sigma$. We have two more constants in the problem, p^+ and E_{LC}. We identify E_{LC} with the energy scale Q_E measured in the NCOS theory

$$E_{LC} = Q_E = -\frac{E}{4\pi\alpha_e}\Sigma\,, \tag{3.4}$$

with E corresponding to a dimensionless measure of energy conveniently introduced in [14]. Given the dispersion relation $2E_{LC}p^+ = Q_E^2$, with $E_{LC} = Q_E + p^y$ and $p^+ = (Q_E - p^y)/2$, this fixes $p^+ = Q_E/2$, which corresponds to $p^y = 0$ in the 'unboosted' frame. Indeed, this is the scenario that was studied in [14], where there was no momentum p^y along the winding string.

The dynamics dictated by (3.2) is then equivalent to a particle in one dimension moving in the potential (see appendix B for details on the background geometry in question)

$$V = \frac{\omega}{2\ G\ \Sigma^2} v \frac{E\sqrt{1+Gv^3} + \sqrt{Gv^3}\,(2\omega \pm E)}{\sqrt{1+Gv^3} \pm \sqrt{Gv^3}} = -\frac{1}{2}\,(\partial_\pm v)^2\ . \tag{3.5}$$

The motion of the closed string has the same qualitative features as that of [14]. In particular, there are identical bounded and scattering processes; and for bounded dynamics, the turning points are $v = 0$ and $v = v_c$ given by

$$Gv_c^3 = \frac{E^2}{4(\omega^2 \pm \omega E)}\ . \tag{3.6}$$

as in [14]. This agreement is a check on the consistency of the formalism and normalizations; we have one arbitrary scale but two conditions to satisfy and, after determining (3.3), a solution is possible.

Applying the Bohr-Sommerfeld quantization for the bounded dynamics scenario

$$\mathcal{I} = \int d\tau d\sigma\ \frac{p^+}{\Sigma} \partial_\mp y^- = \pi N\,, \tag{3.7}$$

we obtain the statement

$$\frac{p^+\omega}{\pi}\Delta\tau = N \qquad \text{for} N \gg 1\,, \tag{3.8}$$

where $\Delta\tau$ is the proper time for a bounce, and N is the level number. Finding the spectrum then involves integrating (3.5) for $\Delta\tau$ between the turning points $v = 0$ and $v = v_c$, which is now slightly more delicate. The following chain of change of variables are very useful

$$Gv^3 = \sinh^2 t\,; \tag{3.9}$$

followed by

$$e^t = \sqrt{y+1}\,. \tag{3.10}$$

The integral's bounds are

$$y_1 = 0\,, \qquad y_2 = \begin{cases} \dfrac{-E}{\omega + E} > 0 & \text{for positive winding} \\ \dfrac{-E}{\omega} > 0 & \text{for negative winding} \end{cases} \tag{3.11}$$

We then find the proper times for positive and negative winding orientations given by

$$\Delta\tau^+ = \frac{a_0 l_e}{|E|}\left(\frac{|E|G}{E+\omega}\right)^{1/3}\left({}_2F_1\left(-\frac{5}{6},\frac{1}{3},\frac{5}{6},\frac{E}{E+\omega}\right) + {}_2F_1\left(\frac{1}{6},\frac{1}{3},\frac{5}{6},\frac{E}{E+\omega}\right)\right); \tag{3.12}$$

$$\Delta\tau^- = \frac{a_0 l_e}{|E|}\left(\frac{|E|G}{\omega}\right)^{1/3}\left({}_2F_1\left(-\frac{1}{3},\frac{1}{3},\frac{5}{6},\frac{E}{\omega}\right) + {}_2F_1\left(\frac{1}{3},\frac{2}{3},\frac{5}{6},\frac{E}{\omega}\right)\right), \tag{3.13}$$

where ${}_2F_1$ is a well-known Hypergeometric function, and

$$a_0 \equiv \frac{2^{11/3}\pi^{3/2}\Gamma(4/3)}{\Gamma(5/6)}\,. \tag{3.14}$$

The spectrum then becomes

$$l_e Q_E \simeq \frac{N^3}{G\omega^2}\frac{\alpha_e}{\Sigma^2} \qquad \text{for positive winding}\,; \tag{3.15}$$

and

$$l_e Q_E \simeq \frac{N^{3/2}}{\sqrt{G\omega}}\sqrt{\frac{l_e}{\Sigma}} \qquad \text{for negative winding}\,; \tag{3.16}$$

with $N \gg 1$. These are indeed identical to the results of [14], even though the steps which we used to arrive at it are slightly different. We conclude that, despite the presence of RR fluxes, we are able to compute part of the strong coupling spectrum of the two dimensional NCOS theory to leading order in inverse D-string charge $1/M$.

It is instructive to briefly look at the mass terms (1.5) for this NCOS geometry. The geometry depicted in figure 1 exhibits an interesting throat region encoded in the metric and, most interestingly, in the axion field. Indeed, when one substitutes the geometry in question into (A.1) and (A.2) and truncates to the infalling string ansatz, one finds that only two terms are non-zero: one proportional to axion flux $\partial\chi$ in (A.1), and one proportional to axion-NSNS-B-field term ${N^{-+}}_a$ in (A.2). Both terms are proportional to the same spinor combination $\bar{\theta}\rho^0\rho^1\theta$. It is then most likely that the effect of the throat would be felt on the worldsheet through the worldsheet spinors becoming massive or massless as the string passes through the region $Gv^3 \sim 1$. Correspondingly, the excited spectrum of the closed string would very much be sensitive to the throat.

4. Strings on AdS and PP-wave backgrounds

We next look at another mechanism by which the quartic interaction terms may disappear from the dynamics. The starting point is to consider a closed string moving in the vicinity of a stack of D3 branes. The $AdS_5 \times S^5$ D3 brane geometry in the coordinates used by [21] appears as

$$ds^2 = R^2\left(\cosh^2\rho\, dt^2 - d\rho^2 - \sinh^2\rho\, d\Omega_3^2 - \cos^2\theta\, d\psi^2 - d\theta^2 - \sin^2\theta\, d{\Omega'}_3^2\right)\,; \tag{4.1}$$

with a five-form RR flux given by

$$G_{t\rho\theta_1\phi_1\phi_2} = 4R^4\cosh\rho\,\sinh^3\rho\sqrt{|g_{\Omega_3}|}\,, \qquad G_{\theta\psi\theta_2\phi_3\phi_4} = 4R^4\cos\theta\,\sin^3\theta\sqrt{|g_{{\Omega'}_3}|}\,. \tag{4.2}$$

The coordinates θ_1, ϕ_1, and ϕ_2 parameterize the three-sphere Ω_3 of volume $\sqrt{|g_{\Omega_3}|}$; and θ_2, ϕ_3, and ϕ_4 parameterize the three-sphere ${\Omega'}_3$ of volume $\sqrt{|g_{{\Omega'}_3}|}$. We separate the index structure for the two spaces we are dealing with: $a, b, \cdots$ shall refer to tangent space indices in AdS_5; while indices $d, e, \cdots$ shall refer to tangent space indices on the S^5. We hope that there will be no confusion between the θ_1 and θ_2 used as spacetime coordinates and the spinors in the worldsheet theory labeled perversely with the same letters. We then write

$$R_{ab} = \frac{4}{R^2}\eta_{ab}\,, \qquad R_{de} = -\frac{4}{R^2}\eta_{de}\,. \tag{4.3}$$

We summarize our notation:

$$\{e^a_m\} \sim \left\{e^0_t, e^1_\rho, e^2_{\theta_1}, e^3_{\phi_1}, e^4_{\phi_2}\right\} \in AdS_5 \tag{4.4}$$

and

$$\left\{e^d_m\right\} \sim \left\{e^5_\theta, e^6_\psi, e^7_{\theta_2}, e^8_{\phi_3}, e^9_{\phi_4}\right\} \in S^5\,. \tag{4.5}$$

We are then to substitute (4.1) and (4.2) in (1.1). We choose the light-cone direction to be parallel to the D3 brane worldvolume. In our parameterization, we pick the $0-2$ (i.e. the t and θ_1) directions in tangent space

$$A^\pm = \frac{1}{2}\left(A^0 \pm A^2\right), \tag{4.6}$$

where A^a is some arbitrary background field. After some unpleasant work, we are lead to the following kinetic term (ignoring the bosonic part)

$$\mathcal{I}^{kin}_{AdS} = -\frac{iR}{2\pi\alpha'}\int d\tau d\sigma \left(\frac{\partial_i t}{e^t_0} + \frac{\partial_i\theta_1}{e^{\theta_1}_2}\right)\left(\sqrt{-h}h^{ij}\bar\theta\rho^0\partial_j\theta - \varepsilon^{ij}\bar\theta\rho^1\partial_j\theta\right)\,. \tag{4.7}$$

The mass terms for the spinors are given by

$$\begin{aligned}\mathcal{I}^{(2)}_{AdS} = {} & \frac{iR}{2\pi\alpha'}\int d\tau d\sigma\left(\frac{\partial_i t}{e^t_0} + \frac{\partial_i\theta_1}{e^{\theta_1}_2}\right)\times \\ & \times\Bigg(2\sqrt{-h}h^{ij}\frac{\partial_j x^m}{e^m_d}\left(\bar\theta\rho^0\rho^1\sigma^{134d}\theta\right) - \frac{1}{48}\sqrt{-h}h^{ij}\frac{\partial_j x^m}{e^m_h}\left(\bar\theta\rho^0\rho^1\sigma^{defg}\theta\right)\varepsilon^{S^5}_{hdefg} + \\ & + \frac{\varepsilon^{ij}}{2}\frac{\partial_j x^m}{e^m_c}\left(\bar\theta\sigma^{ab}\theta\right)\varepsilon^{AdS}_{cab} + \frac{\partial_j x^p}{e^p_k}\omega_{k,ab}\left(\sqrt{-h}h^{ij}\left(\bar\theta\rho^0\sigma^{ab}\theta\right) - \varepsilon^{ij}\left(\bar\theta\rho^1\sigma^{ab}\theta\right)\right)\Bigg)\,.\end{aligned} \tag{4.8}$$

$\omega_{k,ab}$ is the connection of the string frame metric, with the index k summed over *all* tangent space indices. $\varepsilon^{S^5}_{5defg}$ is the antisymmetric form on S^5, and ε^{AdS}_{1ab} is the antisymmetric form on the $\rho-\phi_1-\phi_2$ patch. And there are quartic terms in the spinors

$$\mathcal{I}^{(4)}_{AdS} = \frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-h}h^{ij}\left(\frac{\partial_i t}{e^t_0} + \frac{\partial_i\theta_1}{e^{\theta_1}_2}\right)\left(\frac{\partial_j t}{e^t_0} + \frac{\partial_j\theta_1}{e^{\theta_1}_2}\right)\left(\frac{9}{256}\mathcal{T}^{de}{}_{de} - \frac{101}{768}\mathcal{T}^{ab}{}_{ab}\right). \tag{4.9}$$

Note that we have not fixed the worldsheet symmetries to allow for comparison with different conventions. Partly because of this, the action appears elaborate. Another reason is that we used global AdS coordinates. It is more esthetically pleasing to write the action in Poincaré coordinates [15]–[17]. But our motivation is to use it to take the PP-wave limit, and this form is more suitable for this purpose.

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Following [21], we introduce the coordinates

$$
\begin{aligned}
t &= x^{\tilde{+}} + \frac{x^{\tilde{-}}}{R^2}\,; &\qquad \rho &= \frac{r}{R}\,; \\
\theta &= \frac{y}{R}\,; &\qquad \psi &= x^{\tilde{+}} - \frac{x^{\tilde{-}}}{R^2}\,.
\end{aligned}
\tag{4.10}
$$

Note that the directions $\tilde{+}$ and $\tilde{-}$ are different from the light-cone direction $+$ and $-$. This situation arises since our action can be used only if the light-cone chosen in the gamma matrix algebra is parallel to the D3-brane. Whereas the one of interest in the PP-wave limit picks a spacelike cycle on the S^5, a direction transverse to the D3 branes. We then regard (4.10) as simply a convenient coordinate change in the bosonic fields to facilitate the process of taking the PP-wave limit $R \to \infty$ [21].

We substitute the new coordinates in (4.7)-(4.9), and we take the $R \to \infty$ limit; we arrive at the action

$$
\begin{aligned}
\mathcal{I}_{PP} = \frac{iR}{2\pi\alpha'} \int d\tau d\sigma \partial_i x^{\tilde{+}} \Bigg[& \frac{1}{2}\sqrt{-h}\; h^{ij} \left(\bar{\theta}\rho^0\rho^1\sigma^{5789}\theta + 4\bar{\theta}\rho^0\rho^1\sigma^{1634}\theta\right) \partial_j x^{\tilde{+}} - \\
& - \sqrt{-h}h^{ij}\bar{\theta}\rho^0\partial_j\theta + \varepsilon^{ij}\bar{\theta}\rho^1\partial_j\theta \Bigg] + \mathcal{I}_{PP}^{bos}\,,
\end{aligned}
\tag{4.11}
$$

where $\mathcal{I}_{PP}^{bos}$ is the bosonic part, which is rather trivial. Using the self-duality of the gamma matrix σ^{abcde}, we find

$$
*\sigma^{abcde} = +\sigma^{abcde} \Rightarrow \sigma^{-}\sigma^{1634} = \sigma^{-}\sigma^{5789}\,. \tag{4.12}
$$

We then finally get to

$$
\mathcal{I}_{PP} = iRp^{\tilde{+}} \left[-\bar{\theta}\rho^0\partial_\tau\theta + \bar{\theta}\rho^1\partial_\sigma\theta + \frac{5}{2}p^{\tilde{+}}\bar{\theta}\rho^0\rho^1\sigma^{5789}\theta \right] + \mathcal{I}_{PP}^{bos}\,, \tag{4.13}
$$

where we have additionally fixed the worldsheet conformal symmetry by $h^{00} = 1 = -h^{11}$ and $x^{\tilde{+}} = p^{\tilde{+}}\tau$. Hence, the $R \to \infty$ limit truncates away the non-linear terms in $\mathcal{I}^{(4)}$.

Let us look at the mechanism of this simplification a bit closer. In terms of the length scale of the geometry R, the various objects of interest scale as

$$
e^m_{0,1,5,6} \sim \frac{1}{R}\,, \qquad e^m_{2,3,4,7,8,9} \sim 1\,, \qquad G_{abcde} \sim \frac{1}{R}\,, \qquad R_{abcd} \sim \frac{1}{R^2}\,. \tag{4.14}
$$

This means that

$$
V_i^{\pm} \sim R\,. \tag{4.15}
$$

This is the central statement of the scaling limit. It trivially implies that the kinetic term of the fermions scales as R (see equation (1.2)). And looking at the quartic terms (1.6), we can read off it that it scales as R^0. Hence, in the limit $R \to \infty$, after rescaling $\sqrt{R}\theta \to \theta$, the quartic interactions vanish as $1/R^2$ while the kinetic terms are finite.

The important point is that all this actually follows from dimensional analysis and knowing the general form of the action as given in (1.2), (1.5) and (1.6). The five-form

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G_{abcde} and the Riemann tensor R_{abcd} (with tangent space indices) must scale as $1/R$ and $1/R^2$ respectively as this is the only length scale in a maximally symmetric geometry.[6] Similarly, the scaling of $V_i^{\pm}$ follows because of the same reason, *and* the fact that our choice for light-cone directions did not involve either the holographic direction ρ or any coordinate on the five-sphere (see equation (4.10) for why this matters). But in the derivation of the action (1.1) [12], these are anyways needed assumptions from the outset to validate the expansion in superspace. Hence, the disappearance of the quartic terms is a direct consequence of only dimensional analysis and the fact that our geometry has a single length scale; that is given also knowledge of the general form of the action (1.1). In this sense, identifying other interesting scaling limits in other geometries is promising as it may not require looking at any of the details of (1.1).

Finally, it is easy yet considerably more nontrivial to identify which pieces quadratic in the spinors of (1.5) survive the $R \to \infty$ limit. A short inspection of (4.8), and using (4.14), picks out the two terms with four gamma matrix indices and proportional to $\partial_j \psi$.

5. Discussion

Given the complexity of the coupling of the RR fields to closed string dynamics, it is useful to reflect on all possible strategies of tackling this important problem.

- One approach consists of attempting to get rid of the non-linear fermionic terms by choosing an appropriate ansatz for the closed string dynamics, for example by focusing on left or right moving modes. Solving this sector of the theory first, we may hope to include additional vibrational effects in a perturbative expansion around the ansatz.

- We found that taking a scaling limit in a known geometry can be an easy way to construct a simplified action. In particular, we realize that arguments such as the one for the PP-wave geometry may follow from dimensional analysis and the general form of the action (1.1). This is hence an interesting direction to pursue.

- Another approach would look for special backgrounds for which the quartic terms vanish identically (at long wavelengths with respect to α'). This route involves solving directly for background field configurations such that the coefficients of the quartic terms in (1.7) are zero. To illustrate the complexity of this task, let us look at equations (A.9) to (A.13) set to zero, with the supergravity equations of motion in mind. We may succeed in finding ansatz for which these conditions involve only first derivatives of the fields. We then use a flux configuration that satisifies the conditions to determine the corresponding spacetime geometry. This approach appears rather involved and contrived; but certainly a possible systematic strategy.

- Another hope for computing with (1.1) would be to look for background configurations that yield integrable non-linear sigma models. Indeed, the fermionic part of our

[6] The term proportional to DG in (A.13) is zero since the five-form field strength is covariantly constant when expressed in tangent space.

action, being at most quartic in the spinors, may be imagined to metamorphose, in certain special cases, into Gross-Neveu models [13], with various symmetry structures for the quartic interactions. The integrability of the bosonic sector is somewhat easier to establish (see for example [24]). The guiding principle here is to find a known candidate integrable system, and using its symmetries on the worldsheet, guess at the corresponding background geometry using the form of (1.7). We may then hope to find examples of open/closed string duality such that both sides of the correspondence are exactly solvable worldsheet theories, and the holographic duality amounts to an elaborate 'change of coordinate' or map from one to the other. This is a very interesting open problem that we hope to report on in an upcoming work.

- Lastly, one may try to identify a possible non-trivial vacuum for the spinors for a given background geometry to leading order in a semiclassical approximation. This is a rather complicated problem and it appears the fruitfulness of this approach rests in the particular special features of the background geometries one is to consider.

In summary, the most computationally tractable prospects for learning from the action (1.1) involve restricting closed string dynamics to certain subsets of the general dynamics; subsets or ansatz that circumvent the complicated problem of understanding the effect of the quartic fermionic interactions. Short of this, one needs to hunt for exactly solvable systems or cascades of new scaling limits that progressively simplify the problem. It would also be interesting to study fluctuations of the closed string about the center of mass motion for the NCOS case studied here. In particular, this would clarify the role of the interesting profile of the RR axion flux in the geometry.

Note added: when this work was in its final stages, a paper [25] appeared that overlaps with part of the discussion in section 4. [25] rederives action (1.1) using a similar approach to [12] without considering the quartic spinor terms; this is mainly because [25] focuses on PP-wave backgrounds only, for which these terms vanish. In part of section 4, our PP-wave action is obtained in a different approach. We point out that the process of scaling out the quartic terms in the PP-wave limit of the AdS geometry follows from dimensional analysis.

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A. IIB closed string action

In this appendix, we collect the remaining pieces of the action given in (1.1) written in a slightly more convenient notation [12]. This rewriting is straightforward but algebraically involved, and we refrain from presenting any details. In $\mathcal{I}^{(2)}$ given by (1.5), we have

$$\mathcal{A}_a \equiv i\bar{\theta}\rho^1\sigma_a{}^b\theta\ H_b^{(1)-+} - \frac{i}{4}\bar{\theta}\rho^1\sigma^{bc}\theta\ H^{(1)}_{abc} + \frac{i}{2}\bar{\theta}\rho^0\sigma^{bc}\theta\ \omega_{a,bc} - \frac{i}{2}e^{\phi}\bar{\theta}\rho^0\rho^1\theta\ \partial_a\chi -$$

$$
\begin{aligned}
&-\frac{i}{96}e^{\phi}\bar{\theta}\rho^0\rho^1\sigma^{bcde}\theta\ G_{abcde}+\frac{i}{12}e^{\phi}\bar{\theta}\rho^0\rho^1\sigma_a{}^{bcd}\theta\ G^{-+}{}_{bcd}+\\
&+\frac{i}{4}e^{\phi}\bar{\theta}\sigma^{bc}\theta\ N_{abc}-ie^{\phi}\bar{\theta}\sigma_a{}^b\theta\ N^{-+}{}_b
\end{aligned}\tag{A.1}
$$

$$
\begin{aligned}
\mathcal{B}_a \equiv\ & i\bar{\theta}\rho^0\sigma_a{}^b\theta\ H^{(1)-+}_b+\frac{i}{4}\bar{\theta}\rho^0\sigma^{bc}\theta\ H^{(1)}_{abc}-\frac{i}{2}\bar{\theta}\rho^1\sigma^{bc}\theta\ \omega_{a,bc}+\frac{i}{2}e^{\phi}\bar{\theta}\sigma_a{}^b\theta\ \partial_b\chi-\\
&-\frac{i}{2}e^{\phi}\bar{\theta}\sigma^{bc}\theta\ G^{-+}{}_{abc}-\frac{i}{12}e^{\phi}\bar{\theta}\rho^0\rho^1\sigma_a{}^{bcd}\theta\ N_{bcd}+ie^{\phi}\bar{\theta}\rho^0\rho^1\theta\ N^{-+}{}_a
\end{aligned}\tag{A.2}
$$

$H^{(1)}_{abc}$ and $H^{(2)}_{abc}$ are respectively the NSNS and RR three-form field strengths. In particular, we write as well $H^{(A)}=dB^{(A)}$; e^{ϕ} and χ are respectively the dilaton and RR axion fields; G_{abcde} is the RR five-form field strength; and we have also defined

$$
\mathcal{N}_{abc}\equiv \chi H^{(1)}_{abc}+H^{(2)}_{abc}\,. \tag{A.3}
$$

And $\omega_{a,bc}$ is the connection. Note that all indices are in the tangent space and, in practice, need to be converted to spacetime indices with the vielbein before making use of the action. The gamma matrices are 16×16 matrices[7]

$$
\left\{\sigma^a,\sigma^b\right\}=2\eta^{ab} \tag{A.4}
$$

acting in the space spanned by θ_A. And $\sigma^{a_1a_2a_3\cdots a_n}=\sigma^{[a_1}\sigma^{a_2}\sigma^{a_3}\cdots\sigma^{a_n]}$, with antisymmetrization defined as in [12]. Additional properties of the gamma matrices we use may be found in [12, 28]. The light-cone gauge in spinor space is defined by

$$
\sigma^+\theta_A=\frac{1}{2}\left(\sigma^0+\sigma^d\right)\theta_A=0 \quad \text{for} \quad A=1,2\,, \tag{A.5}
$$

d being the chosen light-cone direction. Collecting the spinors in a doublet $\theta\equiv(\theta_1,\theta_2)$, we also define

$$
\bar{\theta}\equiv\theta\sigma^-\rho^0\ , \tag{A.6}
$$

with ρ's defined as

$$
\rho^0=\begin{pmatrix}0&1\\1&0\end{pmatrix};\qquad \rho^1=\begin{pmatrix}0&1\\-1&0\end{pmatrix}, \tag{A.7}
$$

with the two-dimensional Dirac algebra

$$
\left\{\rho^i,\rho^j\right\}=2\eta^{ij}\,. \tag{A.8}
$$

Our ten dimensional spinors θ_A acquire then a worldsheet spinor representation.

In $\mathcal{I}^{(4)}$, given by (1.6), we have

$$
\begin{aligned}
\mathcal{P}\equiv -\frac{1}{256}\Bigg(& e^{2\phi}G^{-+}{}_{abc}G^{-+abc}+\frac{11}{2}e^{\phi}H^{-+T}{}_a\mathcal{M}H^{-+a}+\frac{29}{144}e^{\phi}H^T_{abc}\mathcal{M}H^{abc}+\\
&+32R^{-+-+}\Bigg);
\end{aligned}\tag{A.9}
$$

[7]Note that the metric signature used throughout is $(+--\cdots)$.

$$
\begin{aligned}
\mathcal{Q}^{(1)}_{ab} \equiv{}& \frac{1}{24} e^{2\phi} G^{-+}{}_{cda} G^{-+cd}{}_{b} + \frac{1}{16}\left(\partial_b \phi H^{(1)-+}{}_{a} + e^{2\phi}\partial_b \chi N^{-+}{}_{a}\right) - \\
& - \frac{29}{96} e^{\phi} H^{-+T}{}_{a} \mathcal{M} H^{-+}{}_{b} + \frac{1}{256} e^{\phi} H_{cda} \mathcal{M} H^{cd}{}_{b} + \frac{1}{2} R^{-}{}_{a}{}^{+}{}_{b}\,; \qquad \text{(A.10)} \\
\mathcal{Q}^{(2)}_{ab} \equiv{}& \frac{1}{1536}\Bigg[96 i e^{\phi}\partial_b \phi N^{-+}{}_{a} - 96 i e^{\phi} \partial_b \chi H^{(1)-+}_{a} - 392 i e^{\phi} H^{(1)-+}_{a} H^{(2)-+}_{b} - \\
& - 94 i e^{\phi} H^{(1)}_{cab} H^{(2)-+c} + 94 i e^{\phi} H^{(1)-+}{}_{c} H^{(2)c}{}_{ab} + 25 i e^{\phi} H^{(1)}_{cda} H^{(2)cd}_{b}\Bigg]; \qquad \text{(A.11)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}^{(1)}_{abcd} \equiv{}& \frac{1}{6144}\Bigg[224 e^{2\phi} G^{-+}{}_{ead} G^{-+e}{}_{bc} - 208 e^{2\phi} G^{-+}{}_{eab} G^{-+e}{}_{cd} - 96 \partial_c \phi H^{(1)}_{abd} - \\
& - 96 e^{2\phi} \partial_c \chi N_{abd} - 336 e^{\phi} H^{T}_{bcd} \mathcal{M} H^{-+}{}_{a} + 26 e^{\phi} H^{T}_{eac} \mathcal{M} H^{e}{}_{bd} + \\
& + 33 e^{\phi} H^{T}_{eab} \mathcal{M} H^{e}{}_{cd} + 384 R_{abcd}\Bigg]; \qquad \text{(A.12)}
\end{aligned}
$$

and finally

$$
\begin{aligned}
\mathcal{R}^{(2)}_{abcd} \equiv{}& -i \frac{e^{\phi}}{3072}\Bigg[384 D_a G^{-+}{}_{cdb} + 960 \partial_a \phi G^{-+}{}_{cdb} + 192 \partial_e \phi G^{-+e}{}_{cb} \eta_{ad} + 48 \partial_c \phi N_{abd} - \\
& - 48 \partial_c \chi H^{(1)}_{abd} - 4 H^{(1)-+}{}_{c} H^{(2)}_{abd} + 4 H^{(1)}_{abd} H^{(2)-+}{}_{c} - \\
& - 72 H^{(1)}_{ecd} H^{(2)e}{}_{ab} + 13 H^{(1)}_{eca} H^{(2)e}{}_{db}\Bigg]. \qquad \text{(A.13)}
\end{aligned}
$$

We have defined the matrix [29]

$$
\mathcal{M} \equiv e^{\phi} \begin{pmatrix} \chi^2 + e^{-2\phi} & \chi \\ \chi & 1 \end{pmatrix}, \qquad \text{(A.14)}
$$

and the doublet

$$
H \equiv \begin{pmatrix} H^{(1)} \\ H^{(2)} \end{pmatrix}. \qquad \text{(A.15)}
$$

Note that there are additional quartic terms to the action that we have not written as they are not relevant to the work in this paper. The reader is referred to an update of [12] for the additional terms that may arise in the full action.

While these equation are quite elaborate in form in the most general cases, for concrete examples like the NCOS and AdS backgrounds we study in the text, they do collapse to much simpler forms. And as a general rule, it is well-established that one is not to expect life to be either simple or pleasant.

B. NCOS background geometry

We write in this appendix the NCOS background geometry used in section 3. For more details, the reader is referred to [14]. In the decoupling limit of interest, the string frame metric is given by

$$
ds^2 = \Omega^2 \left(\frac{v^2}{2\pi^2 \alpha_e}\left(dt^2 - dy^2\right) - \frac{dv^2}{v^2} - 4 d\Omega_7^2\right); \qquad \text{(B.1)}
$$

with

$$\Omega^2 \equiv 8\pi\alpha' \sqrt{\frac{G}{v}} \sqrt{1+Gv^3}\,, \tag{B.2}$$

where G is the coupling of the dual NCOS theory, and v is holographic coordinate identified with energy scale through the UV-IR relation. The 2-form gauge fields are given by

$$B^{(1)}_{ty} = \frac{\alpha'}{\alpha_e} G v^3\,; \tag{B.3}$$

$$B^{(2)}_{ty} = -\frac{\alpha'}{\alpha_e} \frac{M}{(32\pi^2 G)^2}\,. \tag{B.4}$$

The dilaton is

$$e^{\phi} = \left(32\pi^2\right)^2 \frac{G^{3/2}}{M} \frac{1+Gv^3}{v^{3/2}}\,. \tag{B.5}$$

And the RR axion is given by

$$\chi = \frac{M}{(32\pi^2 G)^2} \frac{1}{1+Gv^3}\,. \tag{B.6}$$

The y coordinate is compact of size Σ. The NCOS theory is parametrized by G, M, α_e, and Σ. We take M and G large, and consider in particular the scenario $M \to \infty$ with $G > 1$ but finite. This yields to a finite theory where the supergravity computations may be trusted.

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